Category A: Graphical and Zero Concepts (From Exercise 2.1)

1. Question 1 (1 Mark): The graph of a polynomial y = p(x) intersects the x-axis at exactly three points and touches the x-axis at two other distinct points. Find the total number of real zeroes of this polynomial.
2. Question 2 (1 Mark): If one zero of the quadratic polynomial x^2 + 3x + k is 2, find the exact numerical value of k.

Category B: Relationship Between Zeroes and Coefficients (From Exercise 2.2)

3. Question 3 (2 Marks): Find the zeroes of the quadratic polynomial 4x^2 - 4x - 3, and verify the relationship between its zeroes and its coefficients.
4. Question 4 (2 Marks): Form a quadratic polynomial whose zeroes are (3 + square root of 2) and (3 - square root of 2).
5. Question 5 (3 Marks): If alpha and beta are the zeroes of the quadratic polynomial p(x) = 2x^2 - 5x + 7, find the numerical value of (1/alpha) + (1/beta).
6. Question 6 (3 Marks): If alpha and beta are the zeroes of the polynomial x^2 - 6x + k, find the value of k if it is given that 3(alpha) + 2(beta) = 20.
7. Question 7 (3 Marks): If the zeroes of the polynomial x^2 - px + q are double the zeroes of the polynomial 2x^2 - 5x - 3, find the values of p and q.

Category C: Higher-Order Board Exam Standards

8. Question 8 (3 Marks): If one zero of the quadratic polynomial kx^2 + 11x + 42 is two-thirds of the other zero, find the value of k.
9. Question 9 (5 Marks): If alpha and beta are the zeroes of the quadratic polynomial p(x) = x^2 - x - 2, form a new quadratic polynomial whose zeroes are (2(alpha) + 1) and (2(beta) + 1).
10. Question 10 (5 Marks): If the sum of the zeroes of the quadratic polynomial f(x) = kx^2 + 2x + 3k is equal to twice their product, find the non-zero value of k.

Category A: Graphical Interpretation and Consistency (1 Mark Each)

1. Question 1: Find the value of k for which the pair of linear equations 3x + y = 1 and (2k - 1)x + (k - 1)y = 2k + 1 has no solution.
2. Question 2: If the lines given by 3x + 2ky = 2 and 2x + 5y + 1 = 0 are parallel to each other, what is the numerical value of k?

Category B: Algebraic Methods of Solution (2 Marks Each)

3. Question 3: Solve the following pair of linear equations using the substitution method:

   $2x + 3y = 11$

   $2x - 4y = -24$

   Hence, find the value of 'm' for which $y = mx + 3$.
4. Question 4: Solve the pair of linear equations using the elimination method:

   $3x - 5y - 4 = 0$

   $9x = 2y + 7$

Category C: Condition for Solvability (3 Marks Each)

5. Question 5: Find the values of 'a' and 'b' for which the following pair of linear equations has an infinite number of solutions:

   $2x + 3y = 7$

   $(a - b)x + (a + b)y = 3a + b - 2$
6. Question 6: Determine if the pair of equations $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$ is consistent or inconsistent by comparing their coefficients. Represent this situation graphically by stating the nature of the lines.
7. Question 7: For what value of p will the following pair of linear equations have a unique solution?

   $4x + py + 8 = 0$

   $2x + 2y + 2 = 0$

Category D: Real-World Word Problems (5 Marks Each)

8. Question 8: A fraction becomes 1/3 when 1 is subtracted from the numerator and it becomes 1/4 when 8 is added to its denominator. Find the original fraction.
9. Question 9: Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?
10. Question 10: Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu right now?

Step-by-Step Solutions and Answers

Solution 1

- Step: Apply the condition for no solution (parallel lines).
- Explanation: The condition for no solution is (a1/a2) = (b1/b2) not equal to (c1/c2). Here, a1 = 3, b1 = 1, c1 = 1 and a2 = 2k - 1, b2 = k - 1, c2 = 2k + 1.
- Calculation: Set 3 / (2k - 1) = 1 / (k - 1). Cross-multiplying gives 3(k - 1) = 1(2k - 1) which simplifies to 3k - 3 = 2k - 1. Subtracting 2k and adding 3 to both sides gives k = 2.
- Final Answer: The value of k is 2.

Solution 2

- Step: Apply the parallel lines condition.
- Explanation: For parallel lines, a1/a2 = b1/b2. Here, a1 = 3, b1 = 2k and a2 = 2, b2 = 5.
- Calculation: 3/2 = 2k/5. Cross-multiplying gives 2 * 2k = 3 * 5, which means 4k = 15. Therefore, k = 15/4.
- Final Answer: The value of k is 15/4 (or 3.75).

Solution 3

- Step 1: Solve for x and y using substitution.
- Explanation: From the second equation, 2x = 4y - 24, so x = 2y - 12. Substitute this x into the first equation: 2(2y - 12) + 3y = 11, which expands to 4y - 24 + 3y = 11. This simplifies to 7y = 35, so y = 5. Substituting y = 5 back into x = 2y - 12 gives x = 2(5) - 12 = -2.
- Step 2: Find the value of m.
- Calculation: Substitute x = -2 and y = 5 into y = mx + 3. This gives 5 = m(-2) + 3, meaning 2 = -2m, so m = -1.
- Final Answer: x = -2, y = 5, and m = -1.

Solution 4

- Step: Align equations and use elimination.
- Explanation: Rewrite the equations as Equation 1: 3x - 5y = 4 and Equation 2: 9x - 2y = 7. Multiply Equation 1 by 3 to match the x coefficients: 9x - 15y = 12.

- Calculation: Subtract the modified Equation 1 from Equation 2: (9x - 2y) - (9x - 15y) = 7 - 12, which simplifies to 13y = -5, so y = -5/13. Substitute y into Equation 1: 3x - 5(-5/13) = 4, meaning 3x + 25/13 = 4. This gives 3x = 4 - 25/13 = 27/13, so x = 9/13.
- Final Answer: x = 9/13 and y = -5/13.

Solution 5

- Step: Apply the condition for infinitely many solutions.
- Explanation: The condition is a1/a2 = b1/b2 = c1/c2. This gives 2 / (a - b) = 3 / (a + b) = 7 / (3a + b - 2).
- Calculation: From the first two parts: 2(a + b) = 3(a - b) which simplifies to 2a + 2b = 3a - 3b, leading to a = 5b. From the first and third parts: 2(3a + b - 2) = 7(a - b) which simplifies to 6a + 2b - 4 = 7a - 7b, leading to a - 9b = -4. Substitute a = 5b into this equation: 5b - 9b = -4, so -4b = -4, which means b = 1. Since b = 1, then a = 5(1) = 5.
- Final Answer: a = 5 and b = 1.

Solution 6

- Step: Compare the ratios of the coefficients.
- Explanation: Here, a1/a2 = 1/2, b1/b2 = 2/4 = 1/2, and c1/c2 = -4/-12 = 1/3.
- Calculation: Since a1/a2 = b1/b2 but is not equal to c1/c2, the system of equations has no solution.
- Final Answer: The pair of equations is inconsistent. Graphically, they represent two parallel lines that never intersect.

Solution 7

- Step: Apply the unique solution condition.
- Explanation: For a unique solution, a1/a2 must not be equal to b1/b2. Here, a1 = 4, a2 = 2 and b1 = p, b2 = 2.
- Calculation: 4/2 must not equal p/2. This simplifies to 2 must not equal p/2, which means p must not equal 4.
- Final Answer: The equations will have a unique solution for all real values of p except p = 4.

Solution 8

- Step: Set up equations based on the fraction rules.

- Explanation: Let the fraction be x/y. According to the first condition, (x - 1) / y = 1/3, which cross-multiplies to 3x - 3 = y, or 3x - y = 3. According to the second condition, x / (y + 8) = 1/4, which cross-multiplies to 4x = y + 8, or 4x - y = 8.
- Calculation: Subtract the first equation from the second equation: (4x - y) - (3x - y) = 8 - 3, which gives x = 5. Substitute x = 5 into the first equation: 3(5) - y = 3, which means 15 - y = 3, so y = 12.
- Final Answer: The original fraction is 5/12.

Solution 9

- Step: Formulate speed, distance, and time relationships.
- Explanation: Let the speed of the car starting from A be x km/h and the car from B be y km/h (where x is greater than y). When moving in the same direction, relative speed is (x - y). Distance = Speed * Time, so 100 = (x - y) * 5, which simplifies to x - y = 20. When moving towards each other, relative speed is (x + y). So, 100 = (x + y) * 1, which simplifies to x + y = 100.
- Calculation: Add the two equations together: (x - y) + (x + y) = 20 + 100, which gives 2x = 120, so x = 60. Substitute x = 60 into x + y = 100 to get y = 40.
- Final Answer: The speed of the first car is 60 km/h and the speed of the second car is 40 km/h.

Solution 10

- Step: Formulate age relationships based on time shifts.
- Explanation: Let Nuri's present age be x years and Sonu's present age be y years. Five years ago, Nuri was (x - 5) and Sonu was (y - 5). The problem states x - 5 = 3(y - 5), which simplifies to x - 3y = -10. Ten years later, Nuri will be (x + 10) and Sonu will be (y + 10). The problem states x + 10 = 2(y + 10), which simplifies to x - 2y = 10.
- Calculation: Subtract the first equation from the second equation: (x - 2y) - (x - 3y) = 10 - (-10), which simplifies to y = 20. Substitute y = 20 into the second equation: x - 2(20) = 10, meaning x - 40 = 10, so x = 50.
- Final Answer: Nuri is currently 50 years old and Sonu is currently 20 years old.

Step-by-Step Solutions and Answers

Solution 1

- Step: Count total intersections and touching points.
- Explanation: The number of real zeroes of y = p(x) is equal to the total number of points where the graph meets the x-axis. Intersecting three times gives 3 zeroes, and touching two times gives 2 zeroes.
- Calculation: 3 + 2 = 5.
- Final Answer: The total number of real zeroes is 5.

Solution 2

- Step: Substitute the given zero into the polynomial.
- Explanation: Since 2 is a zero of p(x) = x^2 + 3x + k, we must have p(2) = 0.
- Calculation: (2)^2 + 3(2) + k = 0, which becomes 4 + 6 + k = 0, meaning 10 + k = 0, so k = -10.
- Final Answer: The value of k is -10.

Solution 3

- Step 1: Factorise the polynomial to find zeroes.
- Explanation: Set 4x^2 - 4x - 3 = 0. Splitting the middle term: 4x^2 - 6x + 2x - 3 = 0, which factors to 2x(2x - 3) + 1(2x - 3) = 0, giving (2x + 1)(2x - 3) = 0. So, the zeroes are alpha = -1/2 and beta = 3/2.
- Step 2: Verify the coefficients.
- Explanation: Sum of zeroes = (-1/2) + (3/2) = 2/2 = 1. From coefficients, -b/a = -(-4)/4 = 1. Product of zeroes = (-1/2) * (3/2) = -3/4. From coefficients, c/a = -3/4. Both relationships are verified.
- Final Answer: The zeroes are -1/2 and 3/2.

Solution 4

- Step: Use the sum and product of zeroes formula.
- Explanation: Let alpha = 3 + square root of 2 and beta = 3 - square root of 2.
- Calculation: Sum (S) = (3 + square root of 2) + (3 - square root of 2) = 6. Product (P) = (3 + square root of 2) * (3 - square root of 2) = (3)^2 - (square root of 2)^2 = 9 - 2 = 7. The polynomial equation is x^2 - Sx + P = 0.
- Final Answer: The required quadratic polynomial is x^2 - 6x + 7.

Solution 5

- Step: Relate the required fraction to the sum and product rules.
- Explanation: For 2x^2 - 5x + 7, Sum (alpha + beta) = -(-5)/2 = 5/2. Product (alpha * beta) = 7/2.
- Calculation: (1/alpha) + (1/beta) = (alpha + beta) divided by (alpha * beta) = (5/2) divided by (7/2) = 5/7.
- Final Answer: The numerical value is 5/7.

Solution 6

- Step: Set up a system of linear equations using the zero relationships.
- Explanation: From x^2 - 6x + k, we know alpha + beta = 6. We are given 3(alpha) + 2(beta) = 20.
- Calculation: Multiply the first equation by 2 to get 2(alpha) + 2(beta) = 12. Subtract this equation from the given one: (3(alpha) + 2(beta)) - (2(alpha) + 2(beta)) = 20 - 12, which gives alpha = 8. Substituting alpha = 8 back gives 8 + beta = 6, so beta = -2. Now, k = alpha * beta = 8 * (-2) = -16.
- Final Answer: The value of k is -16.

Solution 7

- Step: Find the original zeroes and double them.
- Explanation: For 2x^2 - 5x - 3 = 0, splitting the middle term gives 2x^2 - 6x + x - 3 = 0, which factors to 2x(x - 3) + 1(x - 3) = 0. The original zeroes are 3 and -1/2. Doubling them gives the new zeroes as 6 and -1.
- Calculation: For the new polynomial x^2 - px + q, Sum of new zeroes = 6 + (-1) = 5, so p = 5. Product of new zeroes = 6 * (-1) = -6, so q = -6.
- Final Answer: p = 5 and q = -6.

Solution 8

- Step: Express the zeroes in terms of a single variable.
- Explanation: Let the zeroes be 'a' and '(2/3)a'. For kx^2 + 11x + 42, Sum of zeroes = a + (2/3)a = -11/k, which simplifies to (5/3)a = -11/k, so a = -33/(5k). Product of zeroes = a * (2/3)a = 42/k, which simplifies to (2/3)a^2 = 42/k, so a^2 = 63/k.

- Calculation: Substitute the value of 'a' into the squared equation: (-33 / 5k)^2 = 63/k, which expands to 1089 / (25 * k^2) = 63/k. Multiplying both sides by k gives 1089 / 25k = 63. Therefore, 25k = 1089 / 63, which means k = 1089 / (25 * 63) = 121 / 175.
- Final Answer: The value of k is 121/175.

Solution 9

- Step: Use basic symmetric functions of roots.
- Explanation: For x^2 - x - 2 = 0, we have alpha + beta = 1 and alpha * beta = -2.
- Calculation: New Sum (S) = (2(alpha) + 1) + (2(beta) + 1) = 2(alpha + beta) + 2 = 2(1) + 2 = 4. New Product (P) = (2(alpha) + 1) * (2(beta) + 1) = 4(alpha * beta) + 2(alpha) + 2(beta) + 1 = 4(alpha * beta) + 2(alpha + beta) + 1 = 4(-2) + 2(1) + 1 = -8 + 2 + 1 = -5. The new polynomial format is x^2 - Sx + P.
- Final Answer: The new quadratic polynomial is x^2 - 4x - 5.

Solution 10

- Step: Translate the condition directly into an algebraic formula.
- Explanation: Given f(x) = kx^2 + 2x + 3k. Sum of zeroes = -2/k. Product of zeroes = 3k/k = 3.
- Calculation: The condition states that Sum = 2 * Product, so -2/k = 2 * 3, meaning -2/k = 6. Solving for k gives 6k = -2, so k = -2/6 = -1/3.
- Final Answer: The non-zero value of k is -1/3.
